

Phenomenological study on the $\bar{p}N \rightarrow \bar{N}N\pi\pi$ reactions

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Abstract

We extend our recent phenomenological study of $pN \rightarrow NN\pi\pi$ reactions to the $\bar{p}N \rightarrow \bar{N}N\pi\pi$ reactions for anti-proton beam momenta up to 3.0 GeV within an effective Lagrangian approach. The contribution of $N^*(1440)$ with its $N\sigma$ decay mode is found to be dominant at the energies close to threshold for $\bar{N}N\pi^+\pi^-$ and $\bar{N}N\pi^0\pi^0$ channels. At higher energies or for $p\bar{n}\pi^-\pi^-$ and $\bar{N}N\pi^\pm\pi^0$ channels where $N^*(1440) \rightarrow N\sigma$ mode cannot contribute, large contributions from double- Δ , $\Delta(1600) \rightarrow N^*(1440)\pi$, $\Delta(1600) \rightarrow \Delta\pi$ and $\Delta(1620) \rightarrow \Delta\pi$ are found. In the near-threshold region, sizeable contributions from $\Delta \rightarrow \Delta\pi$, $\Delta \rightarrow N\pi$, $N \rightarrow \Delta\pi$ and nucleon pole are also indicated. Although these results are similar to those for $pN \rightarrow NN\pi\pi$ reactions, the antinucleon-nucleon collisions are shown to be complementary to the nucleon-nucleon collisions and may even have advantages in some aspects. The PANDA/FAIR experiment is suggested to be an excellent place for studying the properties of relevant N^* and Δ^* resonances.

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I. INTRODUCTION

As an interesting field to study baryon spectrum and properties of strong interaction, double pion production in pion-, photo- and electro-induced reactions has been extensively explored [1]. Recently, we have performed a comprehensive theoretical analysis of the double pion production in nucleon-nucleon collisions [2] based on the new data from CELSIUS and COSY experiments in the past few years [3]. It is meaningful to extend the study to the closely related $\bar{p}N \rightarrow \bar{N}N\pi\pi$ reactions, and herein we present the results.

The experimental studies on the $\bar{p}N \rightarrow \bar{N}N\pi\pi$ reactions were mainly performed in the years around 1970 [4–10] with some additions on the $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ channel by the JETSET Collaboration at 1997 [11]. The data were still scarce. On theoretical side, the one pion exchange (OPE) model [4] and Regge pole model [5], focusing on the beam momenta above 3.0 GeV, included the double- Δ excitations only. However, on experimental side, there was an argument about the data of $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ at the beam momenta around 3.0 GeV [6] whether there was contribution from a N^* with mass about 1400 MeV and width about 80 MeV, respectively. Also, the experiment of $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ at the beam momentum of 2.5 GeV [7] claimed an enhancement at a $\Delta\pi$ invariant mass of 1370 MeV. From the modern point of view, these data might show the presence of the Roper resonance $N^*(1440)$ in the $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ channel. As a matter of fact, the $N^*(1440)$ resonance should play essential role in this channel, which can be postulated from our analysis of $NN \rightarrow NN\pi\pi$ reactions [2, 12] where the $N^*(1440)$ was found to be important in the near-threshold region. The $\Delta(1600)$ and $\Delta(1620)$ resonances are also expected to show up in the $\bar{p}N \rightarrow \bar{N}N\pi\pi$ reactions at high energies because they are found to be important to describe the data of $NN \rightarrow NN\pi\pi$ reactions for the beam momenta around 3.0 GeV [2]. Up to now the properties of these resonances are not well established and especially the nature of $N^*(1440)$ is still in controversial [3]. Therefore it is meaningful to examine whether $\bar{N}N \rightarrow \bar{N}N\pi\pi$ reactions could supply us with useful information. Also the $\bar{p}N \rightarrow \bar{N}N\pi\pi$ channels could serve as a complementary place to test and verify the results of $pN \rightarrow NN\pi\pi$ reactions. As we shall demonstrate later, some channels in antinucleon-nucleon collisions may be very suitable to settle down the problems found in nucleon-nucleon collisions.

Our paper is organized as the following. In Sect. II, we present the formalism and ingredients in our calculation. Then we give our numerical results and discussion in Sect. III,

and a brief summary in Sect. IV.

II. FORMALISM AND INGREDIENTS

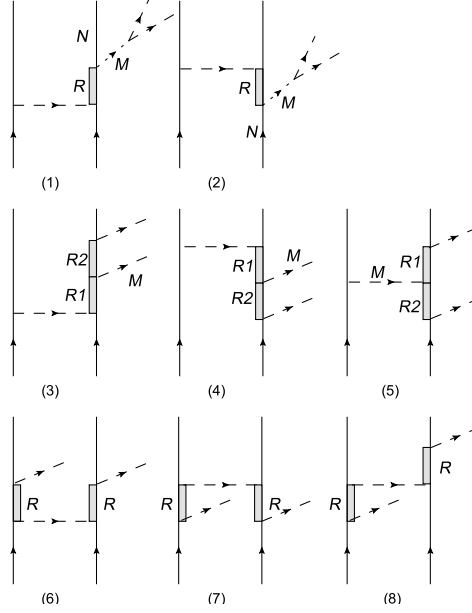


FIG. 1: Feynman diagrams for $\bar{N}N \rightarrow \bar{N}N\pi\pi$. The solid, dashed and dotted lines stand for the (anti)nucleon, mesons and intermediate σ (or ρ)-meson. The shading histograms represent the intermediate baryon resonances or off-shell (anti)nucleon. In the text, we use $R \rightarrow NM$, $R1 \rightarrow R2M$ and double- R to label (1)(2), (3)(4)(5) and (6)(7)(8), respectively.

The Feynman diagrams of $\bar{N}N \rightarrow \bar{N}N\pi\pi$ we considered are depicted in Fig. 1. In the case of $NN \rightarrow NN\pi\pi$ reactions it is needed to symmetrize the initial and final nucleons so there are additional exchanged diagrams, which do not appear in $\bar{N}N \rightarrow \bar{N}N\pi\pi$ channels. The pre-emission diagrams are found to be small in $NN \rightarrow NN\pi\pi$ reactions [2]. Here we include them only for completeness. The formalism and parameters are nearly the same as those used in the study of the $NN \rightarrow NN\pi\pi$. The commonly used Lagrangians for Meson-(anti)Nucleon-(anti)Nucleon couplings [13] are,

$$\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{N} \gamma_5 \gamma_\mu \vec{\tau} \cdot \partial^\mu \vec{\pi} N, \quad (1)$$

$$\mathcal{L}_{\pi \Delta \Delta} = \frac{f_{\pi \Delta \Delta}}{m_\pi} \bar{\Delta}^\nu \gamma_5 \gamma_\mu \vec{\mathcal{K}} \cdot \partial^\mu \vec{\pi} \Delta_\nu, \quad (2)$$

$$\mathcal{L}_{\eta NN} = -ig_{\eta NN}\bar{N}\gamma_5\eta N, \quad (3)$$

$$\mathcal{L}_{\sigma NN} = g_{\sigma NN}\bar{N}\sigma N, \quad (4)$$

$$\mathcal{L}_{\rho NN} = -g_{\rho NN}\bar{N}(\gamma_\mu + \frac{\kappa}{2m_N}\sigma_{\mu\nu}\partial^\nu)\vec{\tau} \cdot \vec{\rho}^\mu N. \quad (5)$$

In the above and following, we explicitly specify the isospin structure of the isospin 3/2 fields. The isospin transition operators \mathcal{I} and \mathcal{K} are defined as,

$$\mathcal{I}_{mn} = \sum_{l=0,\pm 1} (1l\frac{1}{2}n|\frac{3}{2}m)\hat{e}_l^* \quad (6)$$

$$\mathcal{K}_{mn} = \sum_{l=0,\pm 1} (1l\frac{3}{2}n|\frac{3}{2}m)\hat{e}_l^* \quad (7)$$

where m and n are the third components of the isospin projections, and $\vec{\tau}$ the Pauli matrices. At each vertex a relevant off-shell form factor should be used and we take them as [13],

$$F_M^{NN}(k_M^2) = \left(\frac{\Lambda_M^2 - m_M^2}{\Lambda_M^2 - k_M^2} \right)^n, \quad (8)$$

with $n=1$ for π - and η -meson and $n=2$ for ρ -meson. k_M , m_M and Λ_M are the 4-momentum, mass and cut-off parameters for the exchanged meson, respectively. The coupling constants and the cutoff parameters are taken as [13, 14]: $f_{\pi NN}^2/4\pi = 0.078$, $g_{\eta NN}^2/4\pi = 0.4$, $g_{\sigma NN}^2/4\pi = 5.69$, $g_{\rho NN}^2/4\pi = 0.9$, $\Lambda_\pi = \Lambda_\eta = 1.0$ GeV, $\Lambda_\sigma = 1.3$ GeV, $\Lambda_\rho = 1.6$ GeV, and $\kappa = 6.1$. We use $f_{\pi\Delta\Delta} = 4f_{\pi NN}/5$ from the quark model [1]. The mass and width of σ -meson are adopted as 550 MeV and 500 MeV, respectively.

In the $NN \rightarrow NN\pi\pi$ reactions we have shown that the $N^*(1440)$, $\Delta(1232)$, $\Delta^*(1600)$ and $\Delta^*(1620)$ resonances play the major role in the considered energies [2]. Other resonances give negligible contributions so we can safely ignore them. The effective Lagrangians for the relevant resonance couplings are [15, 16],

$$\mathcal{L}_{\pi N\Delta} = g_{\pi N\Delta}\bar{N}\vec{\mathcal{I}} \cdot \partial^\mu \vec{\pi} \Delta_\mu + h.c., \quad (9)$$

$$\mathcal{L}_{\pi NN^*_{(1440)}} = g_{\pi NN^*_{(1440)}}\bar{N}\gamma_5\gamma_\mu\vec{\tau} \cdot \partial^\mu \vec{\pi} N^*_{(1440)} + h.c., \quad (10)$$

$$\mathcal{L}_{\sigma NN^*_{(1440)}} = g_{\sigma NN^*_{(1440)}}\bar{N}\sigma N^*_{(1440)} + h.c., \quad (11)$$

$$\mathcal{L}_{\pi\Delta N^*_{(1440)}} = g_{\pi\Delta N^*_{(1440)}}\bar{\Delta}_\mu\vec{\mathcal{I}} \cdot \partial^\mu \vec{\pi} N^*_{(1440)} + h.c., \quad (12)$$

$$\mathcal{L}_{\pi N\Delta^*_{(1620)}} = g_{\pi N\Delta^*_{(1620)}}\bar{N}\vec{\mathcal{I}} \cdot \vec{\pi} \Delta^*_{(1620)} + h.c., \quad (13)$$

$$\mathcal{L}_{\rho N \Delta_{(1620)}^*} = g_{\rho N \Delta_{(1620)}^*} \bar{N} \gamma_5 \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2} \right) \vec{\mathcal{I}} \cdot \vec{\rho} \Delta_{(1620)}^* + h.c., \quad (14)$$

$$\mathcal{L}_{\pi \Delta \Delta_{(1620)}^*} = g_{\pi \Delta \Delta_{(1620)}^*} \bar{\Delta}_\mu \gamma_5 \vec{\mathcal{K}} \cdot \partial^\mu \vec{\pi} \Delta_{(1620)}^* + h.c., \quad (15)$$

$$\mathcal{L}_{\pi N \Delta_{(1600)}^*} = g_{\pi N \Delta_{(1600)}^*} \bar{N} \vec{\mathcal{I}} \cdot \partial^\mu \vec{\pi} \Delta_{(1600)\mu}^* + h.c., \quad (16)$$

$$\mathcal{L}_{\pi \Delta \Delta_{(1600)}^*} = g_{\pi \Delta \Delta_{(1600)}^*} \bar{\Delta}^\mu \gamma_5 \vec{\mathcal{K}} \cdot \vec{\pi} \Delta_{(1600)\mu}^* + h.c., \quad (17)$$

$$\mathcal{L}_{\pi N_{(1440)}^* \Delta_{(1600)}^*} = g_{\pi N^* \Delta_{(1600)}^*} \bar{N}^* \vec{\mathcal{I}} \cdot \partial^\mu \vec{\pi} \Delta_{(1600)\mu}^* + h.c., \quad (18)$$

For the Resonance-Nucleon-Meson vertices, form factors with the following form are used:

$$F_M^{RN}(k_M^2) = \left(\frac{\Lambda_M^{*2} - m_M^2}{\Lambda_M^{*2} - k_M^2} \right)^n, \quad (19)$$

with $n=1$ for N^* resonances and $n=2$ for Δ^* resonances. We employ $\Lambda_\pi^* = \Lambda_\sigma^* = \Lambda_\eta^* = \Lambda_\rho^* = 1.0$ for $N^*(1440)$, $\Delta(1232)$, $\Delta^*(1620)$ and $\Lambda_\pi^* = 0.8$ for $\Delta^*(1600)$. The Blatt-Weisskopf barrier factors $B(Q_{N^* \Delta \pi})$ are used in the $N^*(1440)$ - Δ - π vertices [17],

$$B(Q_{N^* \Delta \pi}) = \sqrt{\frac{P_{N^* \Delta \pi}^2 + Q_0^2}{Q_{N^* \Delta \pi}^2 + Q_0^2}}, \quad (20)$$

Here Q_0 is the hadron scale parameter, $Q_0 = 0.197327/R$ GeV/c with R the radius of the centrifugal barrier in the unit of fm and chosen to be 1.5 fm to fit the data of $NN \rightarrow NN\pi\pi$ reactions. $Q_{N^* \Delta \pi}$ and $P_{N^* \Delta \pi}$ is defined as,

$$Q_{N^* \Delta \pi}^2 = \frac{(s_N^* + s_\Delta - s_\pi)^2}{4s_N^*} - s_\Delta, \quad (21)$$

$$P_{N^* \Delta \pi}^2 = \frac{(m_{N^*}^2 + m_\Delta^2 - m_\pi^2)^2}{4m_{N^*}^2} - m_\Delta^2, \quad (22)$$

with s_x being the invariant energy squared of x particle. We introduce the Blatt-Weisskopf barrier factors only for $N^*(1440)$ - Δ - π vertices because other resonances, namely $\Delta^*(1600)$ and $\Delta^*(1620)$, begin to contribute at high energies so these factors have little influence on their behavior at the considered energies. On the other hand, as we have addressed, the data of nucleon-nucleon collisions at high energies are scarce so it is meaningful to decrease the adjustable parameters by using fewer form factors. If we would have enough data or go to higher energies it should be certainly necessary to include these form factors in the model.

Because the mass of σ -meson is near the two- π threshold, the following Lagrangians and form factor are employed for the σ - π - π vertex [1, 18],

$$\mathcal{L}_{\sigma\pi\pi} = g_{\sigma\pi\pi} \partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} \sigma, \quad (23)$$

$$\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} \vec{\pi} \times \partial_\mu \vec{\pi} \cdot \vec{\rho}^\mu, \quad (24)$$

$$F_\sigma^{\pi\pi}(\vec{q}^2) = \left(\frac{\Lambda^2 + \Lambda_0^2}{\Lambda^2 + \vec{q}^2} \right)^2, \quad (25)$$

where \vec{q} is the relative momentum of the emitted pion in the center of mass system. We use $\Lambda = 0.8$ GeV and $\Lambda_0^2 = 0.12$ GeV² to normalize this form factor to unity when π - and σ -meson are all on-shell. The decay width of $\sigma \rightarrow \pi\pi$ and $\rho \rightarrow \pi\pi$ yield $g_{\sigma\pi\pi}^2 = 6.06$ and $g_{\rho\pi\pi}^2 = 2.91$.

TABLE I: Relevant parameters used in our calculation. The masses, widths and branching ratios (BR) are taken from central values of PDG [20] except the BR for $N^*(1440) \rightarrow \Delta\pi$.

Resonance	Pole Position	BW Width	Decay Mode	Decay Ratio	$g^2/4\pi$
$\Delta^*(1232)P_{33}$	(1210, 100)	118	$N\pi$	1.0	19.54
$N^*(1440)P_{11}$	(1365, 190)	300	$N\pi$	0.65	0.51
			$N\sigma$	0.075	3.20
			$\Delta\pi$	0.135	4.30
$\Delta^*(1600)P_{33}$	(1600, 300)	350	$N\pi$	0.175	1.09
			$\Delta\pi$	0.55	59.9
			$N^*(1440)\pi$	0.225	289.1
$\Delta^*(1620)S_{31}$	(1600, 118)	145	$N\pi$	0.25	0.06
			$N\rho$	0.14	0.37
			$\Delta\pi$	0.45	83.7

The form factor for the baryon resonance R, $F_R(q^2)$, is taken as,

$$F_R(q^2) = \frac{\Lambda_R^4}{\Lambda_R^4 + (q^2 - M_R^2)^2}, \quad (26)$$

with $\Lambda_R = 1.0$ GeV. The same type of form factors are also applied to the nucleon pole with $\Lambda_N = 0.8$ GeV. The propagators of the exchanged meson, nucleon pole and resonances can be written as [14, 19],

$$G_{\pi/\eta}(k_{\pi/\eta}) = \frac{i}{k_{\pi/\eta}^2 - m_{\pi/\eta}^2}, \quad (27)$$

$$G_\sigma(k_\sigma) = \frac{i}{k_\sigma^2 - m_\sigma^2 + im_\sigma\Gamma_\sigma}, \quad (28)$$

$$G_\rho^{\mu\nu}(k_\rho) = -i \frac{g^{\mu\nu} - k_\rho^\mu k_\rho^\nu / k_\rho^2}{k_\rho^2 - m_\rho^2}, \quad (29)$$

$$G_N(q) = \frac{-i(\not{q} \pm m_N)}{q^2 - m_N^2}. \quad (30)$$

$$G_R^{1/2}(q) = \frac{-i(\not{q} \pm M_R)}{q^2 - M_R^2 + iM_R\Gamma_R}. \quad (31)$$

$$G_R^{3/2}(q) = \frac{-i(\not{q} \pm M_R)G_{\mu\nu}(q)}{q^2 - M_R^2 + iM_R\Gamma_R}. \quad (32)$$

Here " \pm " is for particles and antiparticles, respectively. Γ_R is the total width of the corresponding resonance, and $G_{\mu\nu}(q)$ is defined as,

$$G_{\mu\nu}(q) = -g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu \pm \frac{1}{3M_R}(\gamma_\mu q_\nu - \gamma_\nu q_\mu) + \frac{2}{3M_R^2}q_\mu q_\nu, \quad (33)$$

Because constant width is used in the Breit-Wigner (BW) formula, we adopt the pole positions of various resonances for parameters appearing in the propagators.

The coupling constants appearing in relevant resonances were determined by the empirical partial decay width of the resonances taken from Particle Data Group (PDG) [20], and the detailed calculations of $g_{\rho NR}$ and $g_{\sigma NR}$ from the $R \rightarrow N\rho(\sigma) \rightarrow N\pi\pi$ decay were given in Ref. [21]. The values of cut-off in form factors were adjusted to fit the data of $NN \rightarrow NN\pi\pi$ reactions by hand [2]. Here we would like to mention that in our fit we first determined the cut-off values of Δ and Δ^* resonances by the $pp \rightarrow nn\pi^+\pi^+$ channel which had very small N^* contribution to be much cleaner, and then it was much easier for us to pin down the N^* contributions by a large amount of data in $pp \rightarrow pp\pi^+\pi^-$ and $pp \rightarrow pp\pi^0\pi^0$ channels. We tried to use the same values in the same kind of form factors for all resonances with the aim to reduce the number of free parameters in the model. Take the resonance form factor in Eq. 26 for example, we employed $\Lambda_R = 1.0$ GeV for all the resonances. But in some of form factors, we used different cut-off values for resonances in order to reproduce the data better. For instance, the Λ_π^* for $\Delta^*(1600)$ in Eq.19 was not the same with other resonances. It should be noted that we adopted a nearly half of the decay width of $N^*(1440) \rightarrow \Delta\pi$ in PDG as the recent data of $NN \rightarrow NN\pi\pi$ and $\gamma p \rightarrow p\pi^0\pi^0$ reactions favored [3, 22]. The used decay width of $N^*(1440) \rightarrow N\sigma$ is the same with the value in PDG because we achieved an agreement with the data by adjusting the relevant cut-off parameters. So in our model a larger decay width of $N^*(1440) \rightarrow N\sigma$ compared to PDG was not required. The values of coupling constants and cut-off used in our computation are compiled in Table I,

together with the properties of the resonances and the central values of branch ratios. As we addressed, the parameters in Table I are the same as we used in the analysis of $NN \rightarrow NN\pi\pi$ reactions [2]. So we do not introduce any further free parameters and the calculated results of $\bar{N}N \rightarrow \bar{N}N\pi\pi$ in fact can be viewed as the predictions of our model.

The amplitudes can be obtained straightforwardly by applying the Feynman rules to the diagrams in Fig. 1. Isospin coefficients are considered in different isospin channels. We do not include the interference terms among different diagrams because their relative phases are unknown. The Valencia model seems to show that such terms are very small [12], and our analysis of $NN \rightarrow NN\pi\pi$ reactions also reproduce the data well without including these terms. The multi-particle phase space integration weighted by the amplitude squared can be performed by a Monte Carlo program using the code FOWL from the CERN program library [23].

III. NUMERICAL RESULTS AND DISCUSSION

Fig. 2 and Fig. 3 demonstrate our calculated total cross sections of four isospin channels together with the existing data [8–11]. Our numerical results give an overall good reproduction to all three initial $\bar{p}n$ channels, but overestimate the initial $\bar{p}p$ channel. This may be due to the fact that we have not treated the initial state interactions (ISI) properly. The ISI usually has a weak energy dependence for the meson production processes, so adjusting cutoff parameters in the form factors may partly account for it effectively as for the pp collision [2, 16]. However, while the $\bar{p}n$ is a pure isospin-vector state, the $\bar{p}p$ is a mixture of isospin-scalar and isospin-vector. The annihilation rate for the isospin-scalar $\bar{N}N$ is empirically found to be bigger than the isospin-vector by a factor about 1.7 [24]. The different annihilation rates will cause different ISI reduction factors [25, 26]. This effect is not taken into account in our model calculation. The final state interactions (FSI) may also cause smooth energy-dependent modifications to the total cross sections [27]. Although the ISI and FSI could be taken into account by some more complicated approaches [25–27], they are still of some model dependence. Since in this work we mainly investigate the relative importance of various resonance contributions, we have not included complicated treatments of ISI and FSI which are not expected to influence our main conclusions.

In the following, we shall first address the $\bar{p}n \rightarrow p\bar{n}\pi^-\pi^-$ channel because it is similar to

the $pp \rightarrow nn\pi^+\pi^+$ channel and has negligible N^* contribution to be more clean. Then we shall discuss other three channels. We use the same definitions of various differential cross sections as those used in $NN \rightarrow NN\pi\pi$ reactions [2, 3]. The M_{ij} and M_{ijk} are the invariant mass spectra, and the angular distributions are all defined in the overall center of mass system. The values of vertical axis in the presented figures are all arbitrarily normalized. For concreteness we list the definitions of the angular distributions in the following,

- Θ_M : the scattering angle of M ;
- δ_{ij} : the opening angle between i and j particles;
- Θ_i^{ij} : the scattering angle of i in the rest frame of i and j with respect to the beam axis;
- ϑ_i^{ij} : the scattering angle of i in the rest frame of i and j with respect to the sum of momenta of i and j , corresponding to $\hat{\Theta}_i^{ij}$ defined in Ref. [3].

We try to give adequate information to the future measurements so we show a lot of observables predicted by our model in the following. PANDA is expected to install a 4π solid angle detector with good particle identification for charged particles and photons to get the data of differential cross sections with good quality. Then if there are any experimental results in the future we can immediately know whether our model works and which aspect should be improved in view of the shortcomings of our model. Taking $M_{N\bar{N}}$ and $M_{N\pi}$ as examples, we could identify the role of final state interaction and various resonances, respectively. The angular distributions are also useful to identify different contributions, especially some of which may be sensitive to the details of reaction mechanism.

A. The channel of $\bar{p}n \rightarrow p\bar{n}\pi^-\pi^-$

For this channel, the $N^* \rightarrow N\sigma$ and $N^* \rightarrow \Delta\pi$ cannot contribute. The $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ term is dominant for the energies below 2000 MeV in this channel as shown in Fig. 2(c) and Fig. 3(c). Because the $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ contribution is found to be important to describe the data of various $NN \rightarrow NN\pi\pi$ reactions simultaneously [2], it would be very useful to find other place to get some constrain on this term. The $\bar{p}n \rightarrow p\bar{n}\pi^-\pi^-$ reaction is just an excellent place for such purpose. In Fig. 4 we give the differential cross sections at the beam momentum of 1800 MeV. The peak of invariant mass spectrum $M_{\bar{n}\pi^-}$ is obviously different from that in $pp \rightarrow nn\pi^+\pi^+$ channel which makes it easy for us to identify the $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ term. The steep rise of angular distribution $\Theta_{\bar{n}}$ in forward angle is

distinct from the symmetric shape in $pp \rightarrow nn\pi^+\pi^+$. This is trivial because amplitudes are not symmetric in the exchange of the (anti)nucleons in antinucleon-nucleon collisions.

For the energies above 2300 MeV the $\Delta^*(1600)$ and $\Delta^*(1620)$ terms become significant and it is a good place to study the properties of them. The contribution from $\Delta^*(1600) \rightarrow N^*(1440)\pi$ term begins to take over as the biggest one in this energy region.

As pointed out in our analysis of $NN \rightarrow NN\pi\pi$, the $pp \rightarrow nn\pi^+\pi^+$ is very crucial in determining the cut-off parameters for the form factors of relevant Δ^* resonances due to the fact that this channel has negligible N^* contribution. Unfortunately, the current data of differential cross sections of $pp \rightarrow nn\pi^+\pi^+$ suffer large uncertainties [3] because among its final four particles there are two neutrons which are difficult to detect. Especially, it is hard to figure out that whether there is any dump hump structure or not in $M_{\pi^+\pi^+}$ from the current data of $pp \rightarrow nn\pi^+\pi^+$ [2]. The channel of $\bar{p}n \rightarrow p\bar{n}\pi^-\pi^-$ is just the same as clean as the $pp \rightarrow nn\pi^+\pi^+$ but with only one antineutron in its final four particles, which can be easily reconstructed by the missing mass spectrum. Another ambiguity may rise up from the spectator proton when deuteron target is used to analyze $\bar{p}d \rightarrow p\bar{n}p_{spec}\pi^-\pi^-$, but spectator model is repeatedly confirmed to be reliable in (anti)nucleon-nucleon collisions. So $\bar{p}n \rightarrow p\bar{n}\pi^-\pi^-$ reaction is strongly suggested to be analyzed in PANDA-FAIR and it will be very helpful to distinguish different models.

B. The channel of $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$

This channel is interesting because its double- Δ contribution mainly comes from the simultaneous $\bar{\Delta}^{--}$ and Δ^{++} excitation. As depicted in Fig. 2(a) and Fig. 3(a), for the energies below 1800 MeV the $N^*(1440) \rightarrow N\sigma$ term gives the largest contribution while the nucleon pole and $N \rightarrow \Delta\pi$ terms also influence the near-threshold region significantly. For the energies above 1800 MeV the double- Δ term takes over to be the most important one while $N^*(1440) \rightarrow N\sigma$ and $N^*(1440) \rightarrow \Delta\pi$ rank the second and third, respectively. So unlike the $pp \rightarrow pp\pi^+\pi^-$ and $pp \rightarrow pp\pi^0\pi^0$ channels, in the whole energy region $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ is not suitable to extract the decay widths of $N^*(1440)$ because of the large double- Δ contribution. However, as the best measured channel in antinucleon-nucleon collisions, it is useful to test models. As shown in Fig. 2(a) and Fig. 3(a), our results overestimate the data of the $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ channel for the beam momenta below 2.4 GeV. As discussed

at the beginning of this section, this may be caused by the ISI and FSI which we have not included in our model calculation. The effects from ISI and FSI will not influence much our estimation of relative contributions from various intermediate baryon resonances. This can be checked by the full phase space measurement at PANDA/FAIR.

In Fig. 5 and Fig. 6 we show the calculated differential cross sections at the beam momenta of 1800 MeV and 2200 MeV, respectively. The $N^*(1440) \rightarrow N\sigma$, $N^*(1440) \rightarrow \Delta\pi$ and double- Δ contributions are comparable and have important contributions. Our results are compatible with the old bubble chamber data measured at these energies [8]. Very similar to the NN collisions, the $\pi\pi$ system is sensitive to the change of the contributions as can be seen in the $M_{\pi^+\pi^-}$ and $\cos\vartheta_{\pi\pi}$ spectrums. The double hump structure in $M_{\pi^+\pi^-}$ caused by the $N^*(1440) \rightarrow \Delta\pi$ is obvious. The data of NN collisions did not support these structures [2] but the old bubble chamber data of $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ gave obvious double hump in $M_{\pi^+\pi^-}$ spectrums, especially at the beam momentum of about 1800 MeV [8]. Unfortunately, the statistics was very low and the number of selected events for each beam momentum was at most several hundreds, so the measured results were inconclusive. On theoretical side, the interference terms between $N^*(1440) \rightarrow N\sigma$ and $N^*(1440) \rightarrow \Delta\pi$ might be relevant because their role on the $\pi\pi$ invariant mass distributions have been found in $\pi N \rightarrow \pi\pi N$ [28] and $NN \rightarrow d\pi\pi$ [29], so these terms should be treated with care in the future work. It should be mentioned that this problem may be related to the ABC effect of double-pion production in nuclear fusion reactions [29], so it is meaningful to extensively study it both experimentally and theoretically. The luminosity of PANDA/FAIR is high enough to get the required production rates so the unsettled problem of the $\pi\pi$ system can be further explored in $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ channel.

C. The channel of $\bar{p}n \rightarrow \bar{p}n\pi^+\pi^-$

In Fig. 2(b) and Fig. 3(b), the $N^*(1440) \rightarrow N\sigma$ term is found to dominate in the whole considered energies and the nucleon pole term also gives significant contribution in the near-threshold region. It is worth to point out that unlike the $pn \rightarrow pn\pi^+\pi^-$ channel, the isovector excitation of $N^*(1440)$ in $\bar{p}n \rightarrow \bar{p}n\pi^+\pi^-$ is not enhanced compared to the $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ because charged meson exchange is not allowed in both channels. So in a wide energy region it is suitable to explore the isoscalar excitation of $N^*(1440)$.

The $N^*(1440) \rightarrow \Delta\pi$ term is the second largest for the beam momenta above 1500 MeV and other contributions are much smaller. As can be seen in Fig. 7, the angular distributions of $\vartheta_{\pi^-}^{\pi\pi}$ and $\Theta_{\bar{p}}$ at 1800 MeV are sensitive to the presence of $N^*(1440) \rightarrow \Delta\pi$ term. Though there is possible ambiguity from the spectator proton when deuteron target is used in the experiment, this channel can be a better place to determine the partial decay widths of $N^*(1440)$ than $pp \rightarrow pp\pi^+\pi^-$ and $pp \rightarrow pp\pi^0\pi^0$ where they are complicated by other contributions, such as the double- Δ and nucleon pole terms [2].

D. The channels of $\bar{p}n \rightarrow \bar{p}p\pi^-\pi^0$

The $N^*(1440) \rightarrow N\sigma$ term is not present in this reaction because the σ -meson cannot decay to $\pi^-\pi^0$. The double- Δ term is the most important one in a wide energy range as shown in Fig. 2(d) and Fig. 3(d). The $\Delta \rightarrow \Delta\pi$ and $\Delta \rightarrow N\pi \rightarrow N\pi\pi$ terms have significant contribution below 1600 MeV and also have some contribution at higher energies together with the $\Delta^*(1600)$ and $\Delta^*(1620)$ terms. The agreement with the data is very good.

E. The channels of $\bar{p}p \rightarrow \bar{p}p\pi^0\pi^0$, $\bar{p}n \rightarrow \bar{p}n\pi^0\pi^0$, $\bar{p}p \rightarrow \bar{n}p\pi^-\pi^0$

There are no data on these three channels yet. They can be measured by PANDA experiment. The amplitudes for the $\bar{p}p \rightarrow \bar{p}p\pi^0\pi^0$ and $\bar{p}n \rightarrow \bar{p}n\pi^0\pi^0$ channels are the same except for the difference of $\bar{p}p$ and $\bar{p}n$ FSI. A simultaneous measurement of these two channels may help us to understand the $\bar{p}p$ and $\bar{p}n$ FSI. These two reactions are also similar to the $pp \rightarrow pp\pi^0\pi^0$ reaction except for a different pp FSI and interference between amplitudes by various meson exchanges. Similarly, the $\bar{p}p \rightarrow \bar{n}p\pi^-\pi^0$ reaction has an analogous amplitude with $pn \rightarrow pp\pi^-\pi^0$ reaction except for the $\bar{n}p$ and pp FSI and interference between amplitudes. A simultaneous study of these $\bar{p}N$ and pp reactions may help to pin down contributions from various meson exchanges.

IV. SUMMARY

In this paper, we present an analysis of four isospin channels of the $\bar{p}N \rightarrow \bar{N}N\pi\pi$ reactions for the beam momenta of up to 3.0 GeV within an effective Lagrangian approach. The model

parameters determined from the $NN \rightarrow NN\pi\pi$ reactions are used without introducing any further free parameters. We include contributions from the $N^*(1440)$, Δ , $\Delta^*(1600)$, $\Delta^*(1620)$ and nucleon pole to give a reasonably explanation of the measured total cross sections. The role of the $N^*(1440)$, $\Delta^*(1600)$ and $\Delta^*(1620)$ resonances in $\bar{N}N \rightarrow \bar{N}N\pi\pi$ reactions have never been explored in previous studies. We give some typical differential cross sections which can be tested in the future measurements. We stress that PANDA (anti-Proton ANnihilation at DArmstadt) Collaboration at the GSI-FAIR (Facility of Antiproton and Ion Research) could play an important role in the baryon spectrum and a large amount of events on final states with baryon and antibaryon should be analyzed to extract the properties of relevant resonances. The conclusions reached from our model would be helpful to the future experiments performed at PANDA/FAIR.

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- [1] N. Fettes, V. Bernard and U.-G. Meißner Nucl. Phys. **A669**, 269 (2000); A. Matsuyama, T. Sato and T.-S. H. Lee, Phys. Rep. **439**, 193 (2007); V. Mokeev et al., Phys. Rev. C **80**, 045212 (2009).
 - [2] Xu Cao, Bing-Song Zou and Hu-Shan Xu, Phys. Rev. C **81**, 065201 (2010); Int. J. Mod. Phys. A **26**, 505 (2011).
 - [3] W. Brodowski et al., Phys. Rev. Lett. **88**, 192301 (2002); J. Pätzold et al., Phys. Rev. C **67**, 052202(R) (2003); T. Skorodko, Ph.D. thesis, University of Tübingen, 2009; Abd El-Bary et al., Eur. Phys. J. A **37**, 267 (2008); S. Abd El-Samad et al., Eur. Phys. J. A **42**, 159 (2009).
 - [4] G. Wolf, Phys. Rev. **182**, 1538 (1966).
 - [5] M. Saleem and Fazal-e-Aleem, Prog. Theor. Phys. **70**, 1156 (1983).
 - [6] T. C. Bacon et al., Phys. Rev. Lett. **22**, 43 (1969); T. Ferbel, R. Holmes and S. Stone, Phys. Rev. Lett. **22**, 1141 (1969).

- [7] P. Mason et al., Nucl. Phys. **B30**, 617 (1971).
- [8] J. Lys et al., Phys. Rev. D **7**, 610 (1973).
- [9] P. S. Eastman et al., Nucl. Phys. **B51**, 29 (1973).
- [10] A. Baldini, V. Flamino, W.G. Moorhead and D.R.O. Morrison, Landolt-Börnstein, *Numerical Data and Functional Relationships in Science and Technology*, Vol. 12, Total Cross Sections for Reactions of High Energy Particles, ed. by H. Schopper (Springer-Verlag, Berlin, 1988).
- [11] JETSET collabation, A. Buzzo et al., Z. Phys. C **76**, 475 (1997).
- [12] L. Alvarez-Ruso, E. Oset and E. Hernández, Nucl. Phys. **A633**, 519 (1998), L. Alvarez-Ruso, Ph.D. thesis, University of Valencia, 1999.
- [13] R. Machleidt, K. Holinde and C. Elster, Phys. Rep. **149**, 1 (1987); R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989);
- [14] K. Tsushima, A. Sibirtsev, A. W. Thomas and G. Q. Li, Phys. Rev. C **59**, 369 (1999), Erratum-ibid. C **61**, 029903 (2000).
- [15] Bing-Song Zou and F. Hussain, Phys. Rev. C **67**, 015204 (2003).
- [16] Zhen Ouyang, Ju-Jun Xie, Bing-Song Zou and Hu-shan Xu, Nucl. Phys. **A821**, 220 (2009); Zhen Ouyang, Ju-Jun Xie, Bing-Song Zou and Hu-shan Xu, Int. J. Mod. Phys. **E18**, 281 (2009).
- [17] Bing-Song Zou and D. V. Bugg, Eur. Phys. J. A **16**, 537 (2003).
- [18] C. Schütz, J. Haidenbauer, J. Speth and J. W. Durso, Phys. Rev. C **57**, 1464 (1998).
- [19] Jia-Jun Wu, Zhen Ouyang and Bing-Song Zou, Phys. Rev. C **80**, 045211 (2009).
- [20] The Review of Particle Physics, C. Amsler et al., Phys. Lett. **B667**, 1 (2008).
- [21] Ju-Jun Xie, Colin Wilkin and Bing-Song Zou, Phys. Rev. C **77**, 058202 (2008).
- [22] A. V. Sarantsev et al., Phys. Lett. **B659**, 94 (2008).
- [23] <http://cernlib.web.cern.ch/cernlib/libraries.html>
- [24] M. P. Locher and B. S. Zou, Z. Phys. A **341**, 191 (1992).
- [25] V. Baru et al., Phys. Rev. C **67**, 024002 (2003); C. Hanhart and K. Nakayama, Phys. Lett. **B 454**, 176 (1999).
- [26] J. Haidenbauer et al., Phys. Rev. C **45**, 931 (1992); **46**, 2158 (1992).
- [27] A. Sibirtsev, J. Haidenbauer and U. G. Meissner, Phys. Rev. Lett. **98**, 039101 (2007).
- [28] H. Kamano and M. Arima, Phys. Rev. C **73**, 055203 (2006).
- [29] L. Alvarez-Ruso, Phys. Lett. **B452**, 207 (1999).

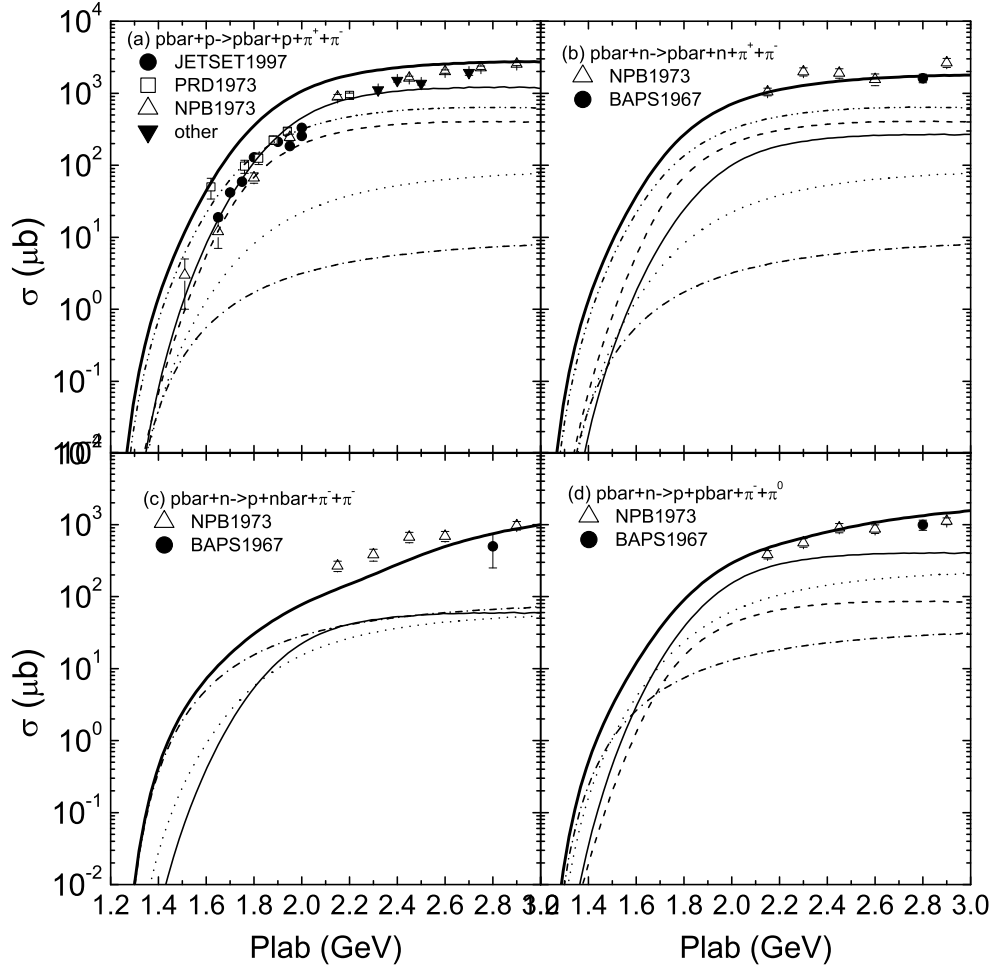


FIG. 2: Total cross sections of $\bar{N}N \rightarrow \bar{N}N\pi\pi$. The solid, dash-dot-dotted, dashed, dotted, dash-dotted, and bold solid curves correspond to contribution from double- Δ , $N^*(1440) \rightarrow N\sigma$, $N^*(1440) \rightarrow \Delta\pi$, $\Delta \rightarrow \Delta\pi$, $\Delta \rightarrow N\pi$, and the full contributions, respectively. The data are from Refs.[7–11].

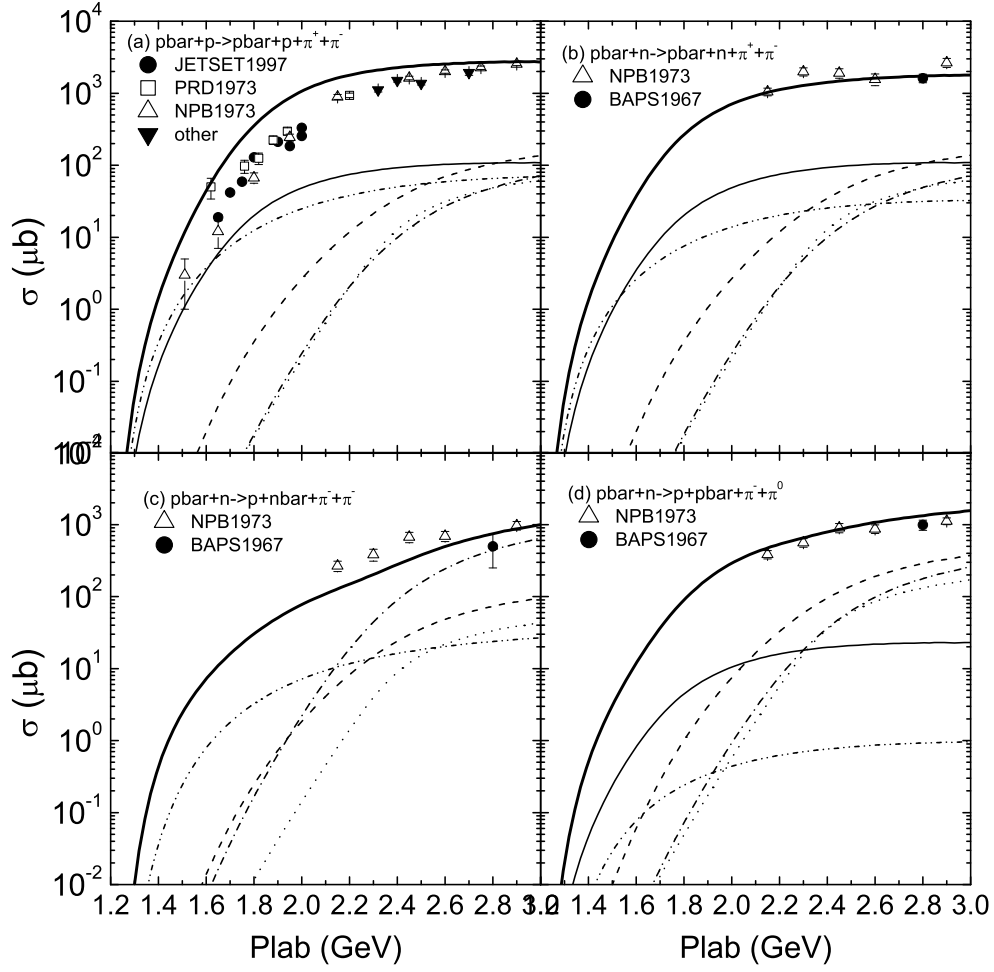


FIG. 3: Total cross sections of $\bar{N}N \rightarrow \bar{N}N\pi\pi$. The dashed, dash-dotted, dotted, dashed-dot-dotted, solid, and bold solid curves correspond to contribution from $\Delta^*(1600) \rightarrow \Delta\pi$, $\Delta^*(1600) \rightarrow N^*(1440)\pi$, $\Delta^*(1620) \rightarrow \Delta\pi$, nucleon pole, $N \rightarrow \Delta\pi$, and the full contributions, respectively. The data are from Refs.[7–11].

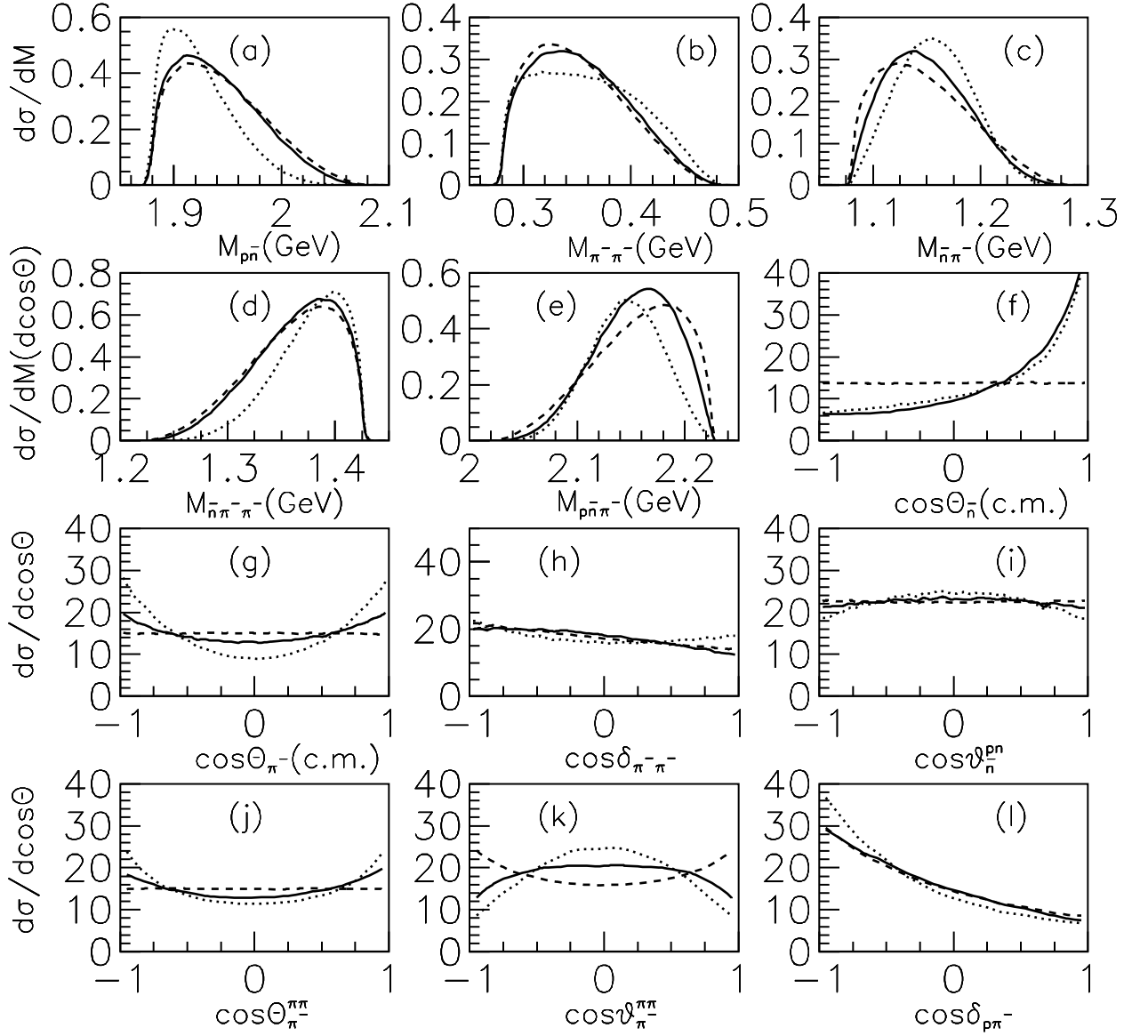


FIG. 4: Differential cross sections of $\bar{p}n \rightarrow p\bar{n}\pi^-\pi^-$ at beam momentum 1800 MeV. The dashed, dotted and solid curves correspond to the phase space, double- Δ and full model distributions, respectively.

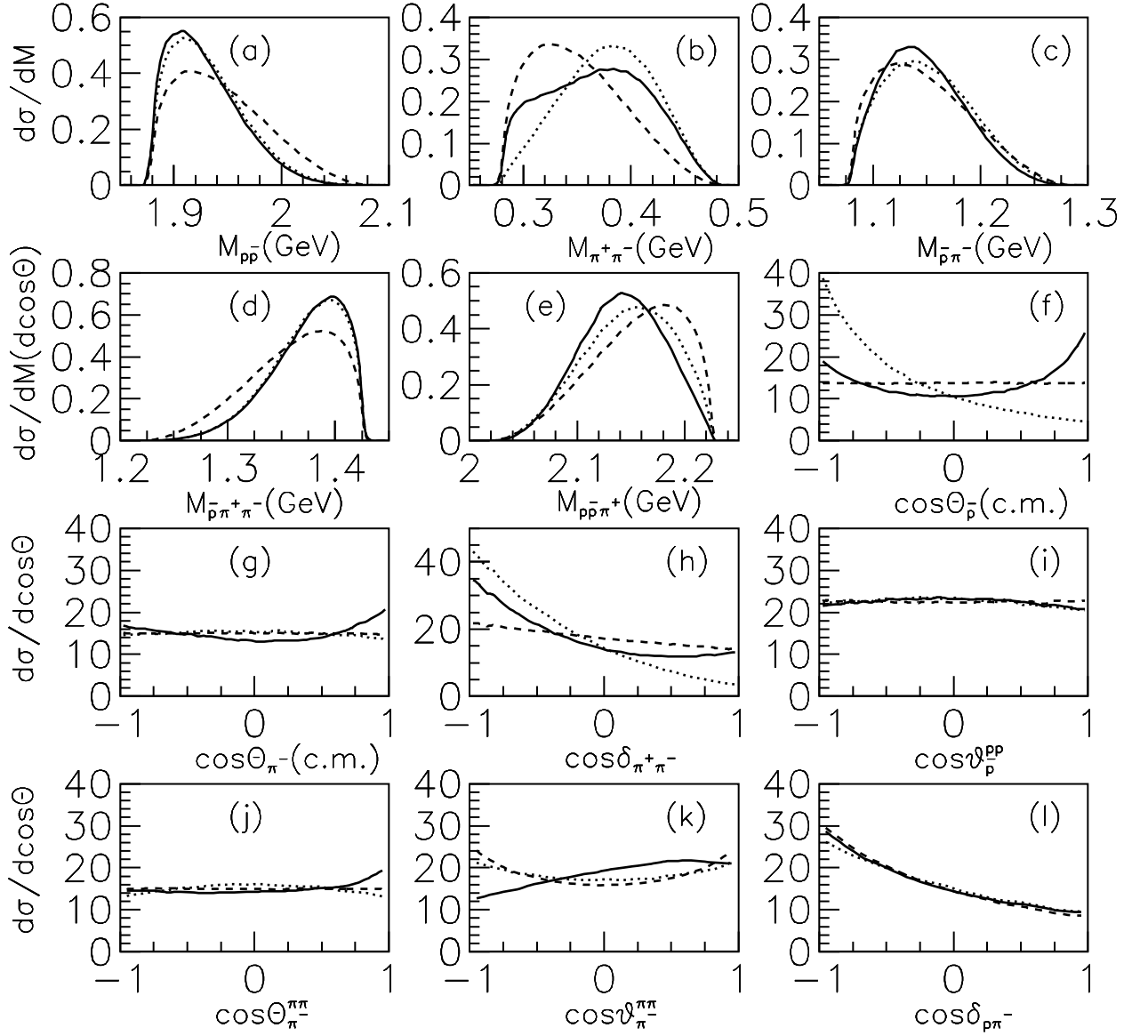


FIG. 5: Differential cross sections of $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ at beam momentum 1800 MeV. The dashed, dotted and solid curves correspond to the phase space, $N^*(1440) \rightarrow N\sigma$ and full model distributions, respectively.

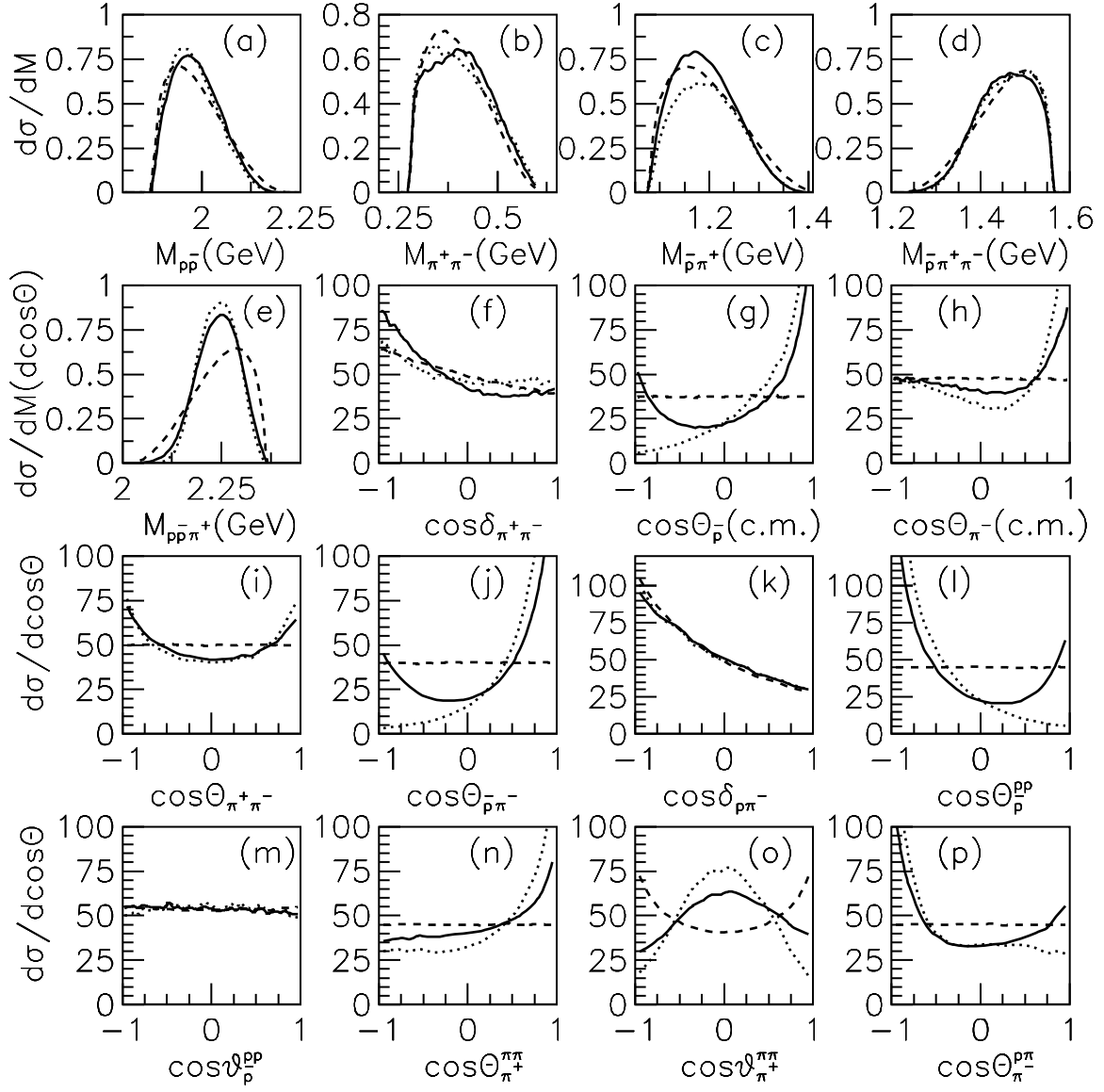


FIG. 6: Differential cross sections of $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$ at beam momentum 2200 MeV. The dashed, dotted and solid curves correspond to the phase space, double- Δ and full model distributions, respectively.

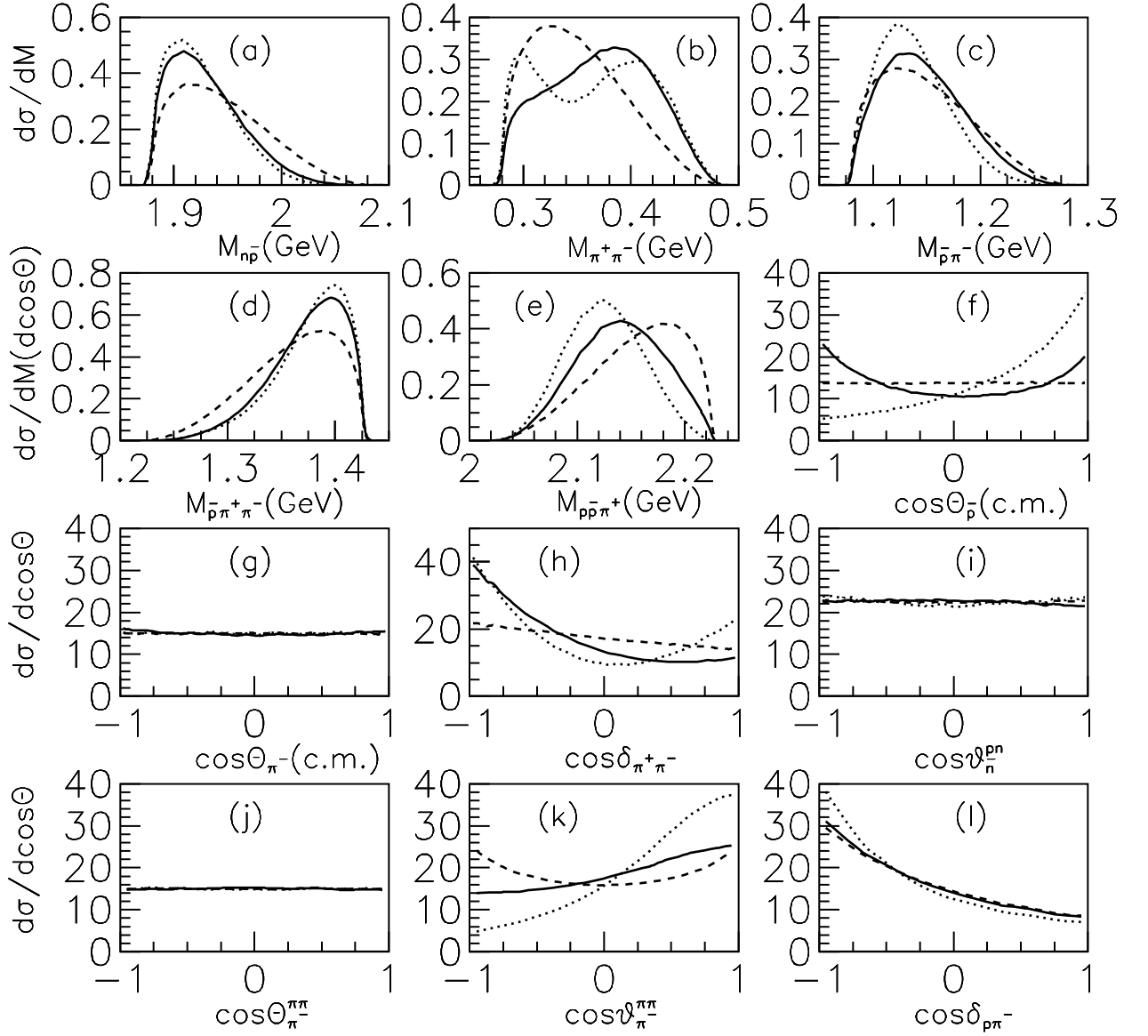


FIG. 7: Differential cross sections of $\bar{p}n \rightarrow \bar{p}n\pi^+\pi^-$ at beam momentum 1800 MeV. The dashed, dotted and solid curves correspond to the phase space, $N^*(1440) \rightarrow \Delta\pi$ and full model distributions, respectively.